**Excitations of interacting e’s in B field**

Now let’s look at the excitations. We’ll recall our Hamiltonian was:



where,



But we’re not actually going to specifically utilize the 2nd quantized form of H. Rather we’ll take the approach employed in the study of the 3He, and 4He fluids. We’ll try to approximate the ground state wavefunction, and the low lying excitations. This approach is relevant, because we’ll be interested in applications to the Quantum Hall Effect, especially in magnetic fields strong enough that all the electrons would be in the lowest Landau level, i.e., filling factors ν ≤ 1.

**Ground state (many-body) wavefunction in non-interacting limit**

We’ll start in the non-interacting limit. So let’s just say we have a free-particle Hamiltonian H0. And we’ll use the symmetric gauge **A** = -(1/2)**r**×**B**. In the QM folder/Time-Independent/particle in B field Symmetric Gauge, we found the eigenstates/energies to be given by:



Let’s suppose that we have high enough B so that all particles are in the lowest Landau level. This lowest energy state would be the one for which η = 0, mℓ = negative integers (we’re assuming electrons here). And this would give us:



Note that the energy is ω/2 – gμBsB, where s = 1/2, since mℓ = ½ if negative charge, and -1/2 if positive charge. And we’ll note an interesting happenstance that we can write this quite simply in complex coordinates z = ξeiφ.



where Cm\_ℓ is the normalization factor. Now we’ll consider all the electrons together. In a strong magnetic field, in the ground state, these should be aligned spin down (magnetic moment therefore *up*) by the magnetic field. Therefore each electron must occupy a different Lz orbital angular momentum m-state. The symmetrized wave function for N electrons would be the Slater determinant of orbital wavefunctions:



where χ↑ is the same spin up spinor (0 1). This can be written as:



which can also be written as:



Just write out the determinant above and you’ll see this is what you get. A shortcut is to recognize that the determinant must be zero when any of the rows are the same, i.e. when zi = zj. And so the determinant must be a polynomial that obeys this property. The only one is that form above. We are somewhat familiar with this. What is the ground state energy?



We can calculate some properties of our Ground State wavefunction. First let’s consider the density:



In the large N limit, this is approximately,



Recalling the Taylor series remainder formula,

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where c is some value between a and x. So we can see this approximation will be okay as long as the remainder is much smaller than the function, i.e.,



Guess I’ll take ln of both sides, and use Stirling’s approximation ln(N!) = NlnN – N.



This condition is the same as:



So apparently our result for the density is good out to a radius that encompasses about half the particles, conservatively. Here’s another way to write it:



And this says our density expression is good out to a radius which encompasses an area equal to ν/2 of the total sample area. Again, this is a conservative estimate. But note that our density result is the actual result when ν = 1, since at that filling, we have:



So basically,



I suppose outside this radius, n exponentially drops or something. We can calculate the pair correlation function g(r) (see QM/Identical Particles/Many-Body Correlations). Recall this is defined via:



where f(**r**,**r´**) is the probability distribution function for the locations of two particles, while g is the pair correlation function. I’m going to also restrict our attention to the region where n(r) ≈ n0 = |e|B/2π. So,



Proceeding,



In the last we can switch i and j on the second two terms to duplicate the first two terms. Filling in the wavefunction, where z = x + iy,



Filling in z = x + iy, and |z| = ξ, etc.,



The sums run from 0 to N-1. As surmised above, I think that as long as we’re away from the edges, we can effectively run this to ∞, and perform the summation.



Noting that 1/2πℓB2 = n0, and then using ξ = r/ℓB, we finally have:



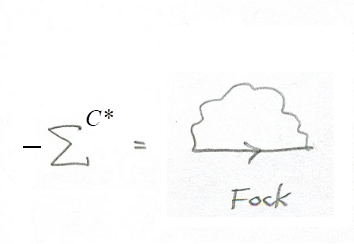
We can use this to calculate the expectation of the electric potential energy between the electrons in the ground state. If we revisit the Nearly Free Electrons folder/Ground State Energy, we’ll come across the formula we wrote down for the potential energy of interacting electrons and uniform positive jelly. And it was:



We can fill in our g(r) result and see what we get. For simplicity we’ll again restrict our attention to the region where 1/2πℓB2 = n0. Then we have:



Might add that this is equivalent, diagrammatically, to evaluating the self-energy up through the exchange/Fock diagram:



Last let’s get the Structure factor. For simplicity, again, we’ll presume that we’re in the smallish r region such that 1/2πℓB2 = n0.



So we have:



**Ground state (many-body) wavefunction in interacting case**

For large enough B, where all particles are in the ground state, e-e interaction seems to become increasingly important, as B constricts the particles to an ever tightening circle. Looks like for ν = 1, the non-interacting ground state above is a good approximation to the actual ground state. But as ν decreases, it becomes increasingly malapropos. Laughlin proposed an ansatz for the interacting ground state wavefunction:



where C is a normalization constant. Note that unlike the previous one, this wavefunction doesn’t correspond to any HF Slater determinant of independent single particle states. And can see that the p exponent would cause an even greater drop in density as electrons approach each other than did the non-interacting ground state, as befits a repulsive e-e interaction. AS requires that p be an odd integer. The QHE corresponds to p = 1, and the other fractional steps correspond to 1/p. 1/p is the fractional filling level of the state as one can verify if you calculate the density function of this wavefunction. It is this fractional power that makes these guys ‘anyons’ I think.



It’s useful to write this as:



Then we can cast this in the form of a functional – see Stat Mech folder/Classical MFT. Let n(r) be the particle density. Then,



where,



The first two terms in F are sensible. The last term is an ‘entropy’ term which takes acount of the number of ways we can interchange particle coordinates without changing the density (this term is necessary as otherwise each function n(r) wouldn’t get the statistical weight it deserves). Let’s say we were interested in the mean field density. This would be the one which maximizes n0, or equivalently, minimizes F – well I guess this would really be the most *probable* density, not the average/mean density. Note that the average density ought to be n(r) = ∫d2r2d2r3…d2rN n0(r,r2,…,rN), in accordance with our discussion in the Many Body Correlations file in QM folder/Multiple Particles. But we do expect these to be the same for all intents and purposes. So we’ll take a functional derivative of F w/r to n(r) and set to zero.



I’m presuming n doesn’t depend on φ – don’t see how it would. Let’s take a derivative w/r to r,



And we can change variables φ´ → φ´ + φ, and get:



Can do a little more simplification,



So now,



Can use Calculus of Residues to get this integral. Let z = eiφ. Then,



So,



And finally, our density equation comes to:



We can make some more progress by differentiating this w/r to r,



So we’ve reduced our integral equation for the density to a differential equation. The density ought to be constant until we get out near the edges. So up till then, the n´(r) shouldn’t matter, and we end up with:



We’ll recall the non-interacting density was just 1/2πℓB2, while this is ν/2πℓB2. Since ν < 1, this means that the density is lower than the non-interacting density, for the same B. This makes sense this the electrons should repell each other.

**Low Lying Excitations**

The excitation of wave vector k is obtained by multiplying the GS wave function, Ψ0, by the density fluctuation which gives the excited state Ψk. This is just the sort of excitations that we used before (will use), when analyzing the liquid He condensate in the Quantum Fluid folder, and when analyzing the Fermi liquid.



where,



(recognizing that n(k) = Σjeikr and it’s ascertained the factor Lk = 1/√NS(k) through imposing normalization. As usual, we determine the excitation energy by,



and so we eventually get (see Quantum Fluids folder):



where εk = k2/2m. For the case where ν = 1, we have,



At large k the excitations go to the free particle kinetic energy. At small values they go to ωc. So these are simply single particle excitations. One suspects that we can pile on excitations, as was done for the superfluids, to get?



Another low energy excitation has been found which are Magnetorotons – collective modes, which he discusses but I’m not.